Name ______________________________

MATH 280 Multivariate Calculus Spring 2011 Exam #5

Instructions: Do your work on separate paper. You can work on the problems in any order. Clearly label your work on each problem with the problem number. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem. (100 points total)

A. Pay attention to the statement of each problem. If a problem asks you to "Set up a definite/iterated integral or integrals..." you do not need to evaluate the integral(s).

B. For full credit, each definite integral you set up should be expressed entirely in terms of one variable and each iterated integral you set up should be expressed entirely in terms of one pair of variables.

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1. (a) Use the grid on the accompanying sheet to sketch the vector field $\vec{F} = y\,\hat{\imath} + x\,\hat{\jmath}$. Plot an output for each of the points provided on the grid. (4 points)

   (b) Sketch the curve $y = x^3$ for $x = -1$ to $x = 1$ on the same plot below. (1 points)

   (c) Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}$ as given in (a) and C as the curve in (b). (10 points)

   (d) Explain why your result in (c) is reasonable in relation to your plot from (a) and (b). (3 points)

2. (a) Show that $\vec{F} = \left(y^2z+1\right)\hat{\imath} + \left(2xyz+z\right)\hat{\jmath} + \left(xy^2+y\right)\hat{k}$ has a potential function without explicitly finding a potential function. (4 points)

   (b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}$ is given in (a) and C is any curve that starts at $(0, -2, 1)$ and ends at $(2, 1, 3)$. (10 points)

3. Set up an iterated integral or integrals to compute the surface area of the ellipsoid given by $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Note: In terms of spherical coordinate, the ellipsoid can be described as

$$x = a\cos\theta\sin\phi \qquad y = b\sin\theta\sin\phi \qquad z = c\cos\phi.$$

   for $0 \le \theta \le 2\pi$ and $0 \le \phi \le \pi$. (14 points)

4. Set up an iterated integral or integrals equal to the surface integral $\displaystyle\iint_S \vec{F} \cdot d\vec{A}$ where $\vec{F} = y\,\hat{\imath} + x\,\hat{\jmath} + z^2\,\hat{k}$ and S is the piece of the hyperboloid $x^2 + y^2 - z^2 = -1$ for $1 \le z \le 2$ with area element vectors $d\vec{A}$ pointing away from the z-axis. (14 points)

5. (a) Compute the divergence of the vector field $\vec{F} = x^2y\,\hat{\imath} + z\,\hat{\jmath} + z^2\,\hat{k}$ for the point $(3, 1, 2)$. (8 points)

   (b) Give a brief interpretation of your value from (a). (2 points)

6. (a) Compute the curl of the vector field $\vec{F} = x^2y\,\hat{\imath} + z\,\hat{\jmath} + z^2\,\hat{k}$ for the point $(3, 1, 2)$. (8 points)

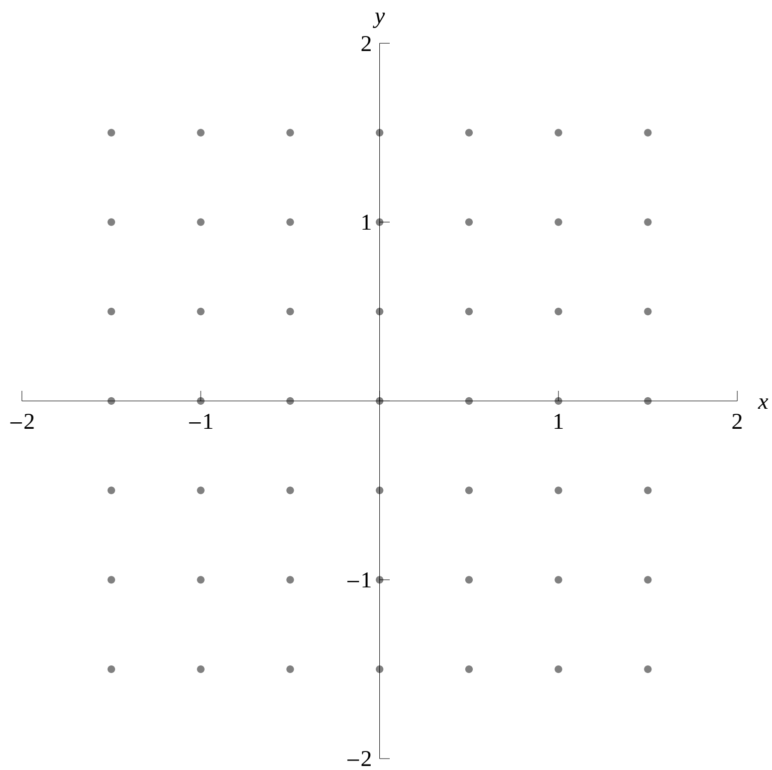
   (b) Give a brief interpretation of your value from (a). (4 points)

7. (a) The conclusion of Stokes' Theorem is $\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{A} = \oint_C \vec{F} \cdot d\vec{r}$. In a brief sentence or two, explain what S and C represent in this statement. (3 points)

   (b) The conclusion of the Divergence Theorem is $\iiint_D (\vec{\nabla} \cdot \vec{F})\, dV = \oiint_S \vec{F} \cdot d\vec{A}$. In a brief sentence or two, explain what D and S represent in this statement. (3 points)

   (c) Briefly describe one similarity between Stokes' Theorem and the Divergence Theorem. (3 points)

8. Use the Divergence Theorem to evaluate $\oiint_S \vec{F} \cdot d\vec{A}$ where $\vec{F} = xy\,\hat{\imath} + xz\,\hat{\jmath} + yz\,\hat{k}$ and S is the surface of the cube in the first octant with edges along the coordinate axes having one corner at $(0, 0, 0)$ and the far corner at $(3, 3, 3)$. (9 points)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